Charge-insensitive qubit design derived from the Cooper pair box

Pan Zhang ,EPFL

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Introduction

Main Contents for this week

- Summary of Transmon qubits
  - Transmon and Effective circuit
  - Charge Dispersion
  - Anharmonicity
- circuit QED
  - System Hamiltonian in Jaynes-Cummings Hamiltonian
  - System Hamiltonian in dispersive limits

Transmon and Effective circuit

From Copper pair box to transmon qubit, the only modification is a shunting connection of the two superconductors via a large capacitance C_B

- Hamiltonian $= 4E_c(\hat{n} - n_g)^2 - E_J cos(\hat{\varphi})$
- $\hat{n}$ is number of cooper piars transferred between the islands.
- $\hat{\varphi}$ is the gauge invariant phase between the superconductors.

Transmon and Effective circuit: Transmon Energy levels

The eigenenergy of hamiltonian is given by solving the stationary Schrodinger equation in phase basis.

- $[4E_C(-i\frac{d}{d\varphi})^2 - E_J cos(\varphi)\psi](\varphi) = E\psi(\varphi)$
- Using Ansatz solution we reduce the differential equation into $g^{''}(x) + (\frac{E}{E_C} + \frac{E_j}{E_C} cos(2x))g(x) = 0$
- We get the $E_m(n_g) = E_C a_{2[n_g+K(m,n_g)]}(-E_j/2E_c)$
- Here, $K(m, n_g)$ is a function that can correctly sorting the eigenvalues. It gives you positive or negative integer.

Transmon and Effective circuit: Transmon[1]

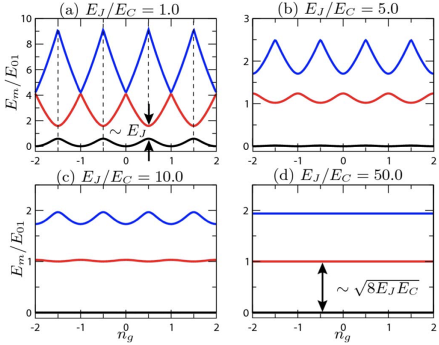


Figure:
$$E_m(n_g) = E_C a_{2[n_g+K(m,n_g)]}(-E_j/2E_c)$$

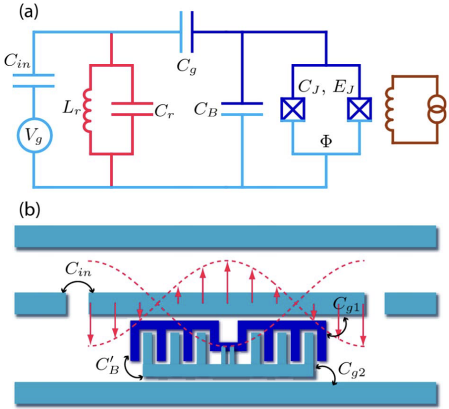


Figure: $4E_c(\hat{n} - n_g)^2 - E_J cos(\hat{\varphi})$

[1] Jens Koch et al. "Charge-insensitive qubit design derived from the Cooper pair box". In: Physical Review A 76.4 (2007), p. 042319. DOI: 10.1103/PhysRevA.76.042319.

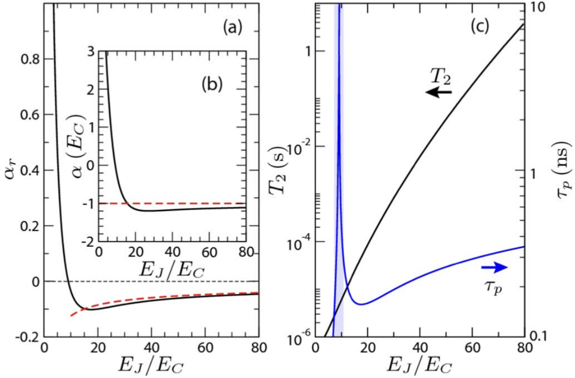
Charge dispersion

- For a 2 level system, we expect a stable energy level.
- At large E_J/E_c ratio, we saw from previous slides, a very stable energy level.
- We employ WKB approximation on energy level, get $E_m(n_g) \approx E_m(n_g = 1/4) - ce^{-\sqrt{8E_J/E_C}} cos(2\pi n_g)$
- the exponential term heavily suppress the charge dispersion resulting in a stable energy gap.

Anharmonicity

- The energy level can be approximate as $E_m \approx -E_J + \sqrt{8E_cE_j}(m+1/2) - \frac{E_c}{12}(6m^2+6m+3)$
- The energy eigenvalue is expanded the cosine(φ) around the fourth order of around $\varphi = 0$.
- and using the quantization relation express $\varphi = (\frac{2E_c}{E_j})^{1/4}(a^\dagger + a)$
- Find the asymptotic expression for relative anharmonicity is $\alpha_r = \alpha/E_{01} \approx -(8E_J/E_C)^{-1/2}$

Anharmonicity[2]



Absolute aharmonicity $\alpha = E_{12} - E_{01}$. relative aharmonicity $\alpha_r = \alpha / E_{01}$. $E_J/E_c \approx 9$ negative relative aharmonicity

[2] Jens Koch et al. "Charge-insensitive qubit design derived from the Cooper pair box". In: Physical Review A 76.4 (2007), p. 042319. DOI: 10.1103/PhysRevA.76.042319.

Circuit QED

Full analysis of Transmon circuit

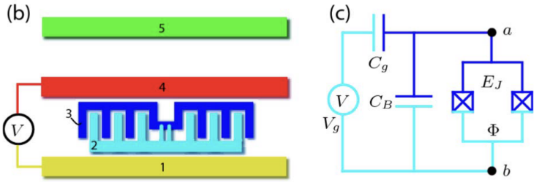
$$\hat{H} = \frac{\hat{\phi}_r^2}{2L_r} + \frac{(C_B + C_g)\hat{Q}_r^2}{2C_*^2}$$
$$+ \frac{(C_g + C_{in} + C_r)\hat{Q}_J^2}{2C_*^2} - E_J \cos\left(\frac{2\pi}{\hbar}\hat{\phi}_J\right)$$
$$+ \frac{C_g\hat{Q}_r\hat{Q}_J}{C_*^2} + \frac{(C_B C_{in} + C_g C_{in})\hat{Q}_r V_g + C_g C_{in}\hat{Q}_J V_g}{C_*^2}.$$

$C_*^2 \approx C_B C_r + C_g C_r$ assuming C_r is much larger than other values. After substitution, we find:

$$\hat{H} = \frac{\hat{\phi}_r^2}{2L_r} + \frac{\hat{Q}^2}{2C_r} + \frac{\hat{Q}_J}{2(C_B + C_g)} - E_J \cos\left(\frac{2\pi}{\hbar}\hat{\phi}_J\right) + \frac{C_{in}\hat{Q}_r V_g}{C_r} \tag{1}$$

Circuit QED[3]

Full analysis of Transmon circuit



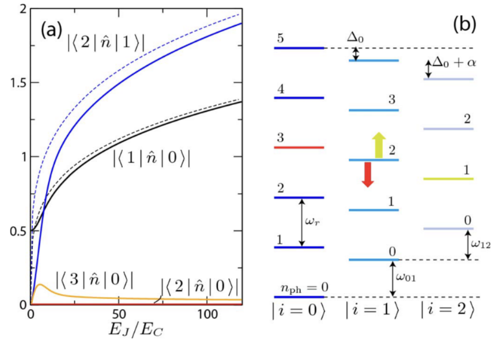
The general JC hamiltonian is optained: $\hat{H} = \hbar \sum_j \omega_j \left|j\right\rangle \left\langle j\right| + \hbar\omega_r \hat{a}^\dagger \hat{a} + \hbar \sum_{i,j} g_{ij} \left|i\right\rangle \left\langle j\right| (\hat{a} + \hat{a}^\dagger)$

[3] Jens Koch et al. "Charge-insensitive qubit design derived from the Cooper pair box". In: Physical Review A 76.4 (2007), p. 042319. DOI: 10.1103/PhysRevA.76.042319.

Circuit QED, Transmon in Jaynes-cummings Hamiltonian

- Rewrite the system in uncoupled transmon states $|i\rangle$
- The Hamiltonian reach $\hat{H} = \hbar \sum_j \omega_j |j\rangle \langle j| + \hbar\omega_r \hat{a}\hat{a} + \hbar \sum_{i,j} g_{ij} |i\rangle \langle j| (\hat{a} + \hat{a}^\dagger)$
- where $\hbar g_{ij} = 2\beta e V^0_{rms} \langle i| \hat{n} |j\rangle$

Circuit QED, Transmon in Jaynes-cummings Hamiltonian[4]



All i+n where $|n| > 1$ term can be effectively ignored in the large $\frac{E_J}{E_C}$ limit.
Only consider the adjacent states in the large E_j/E_climit, we find

$$\hat{H} = \hbar \sum_j \omega_j |j\rangle\langle j| + \hbar\omega_r \hat{a}^\dagger \hat{a} + \hbar \sum_{i,i+1} g_{i,i+1} |i\rangle\langle i+1| \hat{a}^\dagger + H.C.$$

[4] Jens Koch et al. “Charge-insensitive qubit design derived from the Cooper pair box”. In: Physical Review A 76.4 (2007), p. 042319. DOI: 10.1103/PhysRevA.76.042319.

Circuit QED, Dispersive limit

In the dispersive limit, the detunnings $\Delta_i = \omega_{i,i+1} - \omega_r$ between transmon and cavity are large, where coupling strength of the cooper pair between uncoupled transmon state $|0\rangle$ and $|1\rangle$. g_{01} is much smaller than detunning defined above.
The consequence is canonical transformation gauge the cavity-qubit interaction to the lowest order.

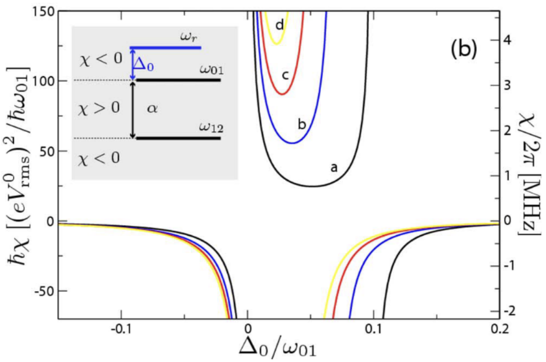
- Hamiltonian after canonical transformation: $\hat{H}_{eff} = \frac{\hbar\omega'_{01}}{2}\hat{\sigma}_z + (\hbar\omega'_r + \hbar\chi\hat{\sigma}_z)\hat{a}^\dagger\hat{a}$ in dispersive limit.
- $\omega'_r = \omega_r - \chi_{12}/2$
- $\omega'_{01} = \omega_{01} + \chi_{01}$
- Define: $\chi = \chi_{01} - \chi_{12}/2$, where $\chi_{ij} = \frac{g_{ij}^2}{\omega_{ij} - \omega_r}$
- Directly compute χ, we find $\hbar\chi \approx -(\beta e V_{rms})^2 (\frac{E_J}{2E_C})^{1/2} \frac{E_C}{\hbar\Delta_0(\hbar\Delta_0 - E_C)}$.

Circuit QED, Dispersive limit

$$\hbar\chi \approx -(\beta e V_{rms})^2 (\frac{E_J}{2E_C})^{1/2} \frac{E_C}{\hbar\Delta_0(\hbar\Delta_0 - E_C)} = g^2(\frac{1}{\Delta_0} - \frac{1}{\Delta_1}), \chi_{ij} = \frac{g_{ij}^2}{\omega_{ij} - \omega_r}, \chi = \chi_{01} - \chi_{12}/2 \tag{2}$$

- Noting the negative sign at beginning of the equation. χ is only positive when $0 < \Delta_0 < E_c$.
- This don't violate the dispersive regime defined as $_{01}/\Delta_0 << 1$ or another word, dispersive regime depend on $(\frac{E_j}{E_c})^{1/4} << \omega_{01} - \omega_r$.
- The strong coupling regime, in the other hand, require $g >> max(\gamma, \kappa)$

Circuit QED, Dispersive limit[5]



The plot of $\chi = \chi_{01} - \chi_{12}/2$. The pole is at $\Delta_0 = 0$ and $\Delta_0 = E_c$

[5] Jens Koch et al. "Charge-insensitive qubit design derived from the Cooper pair box". In: Physical Review A 76.4 (2007), p. 042319. DOI: 10.1103/PhysRevA.76.042319.

Conclusion, Transmon

- Energy level proportional to Mathieu characteristic a value.
  $E_m(n_g) = E_C a_{2[n_g+K(m,n_g)]}(-E_j/2E_c)$
- Charge dispersion is suppressed exponentially
  $E_m(n_g) \approx E_m(n_g = 1/4) - ce^{-\sqrt{8E_J/E_C}} cos(2\pi n_g)$
- Anharmonicty is decreased with weak power law. $\alpha_r = \alpha/E_{01} \approx -(8E_J/E_C)^{-1/2}$

Conclusion, Circuit QED[6]

- Circuit hamiltonian reaches Jaynes-Cummings model after employ rotating wave approximation. $\hat{H} = \hbar \sum_j \omega_j |j\rangle \langle j| + \hbar\omega_r \hat{a}^\dagger \hat{a} + \hbar \sum_{i,i+1} g_{i,i+1} |i\rangle \langle i+1| \hat{a}^\dagger$
- In dispersive limit, where $\frac{g_{01}}{\Delta_{01}} << 1$, the hamiltonian reduce to $\hat{H}_{eff} = \frac{\hbar\omega'_{01}}{2}\hat{\sigma}_z + (\hbar\omega'_r + \hbar\chi\hat{\sigma}_z)\hat{a} \dagger \hat{a}$.
- The qubit frequency is tied with resonator frequency with effective dispersive shift $\chi_{ij} = \frac{g_{ij}^2}{\omega_{ij} - \omega_r}$
- The strong coupling regime is reached when coupling strength $g >>$maximum channel decay rate.
- We find the strong coupling can be achieved in dispersive regime gives oppotunities of performing quantum nondemolition.

[6] Alexandre Blais et al. "Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation". In: Physical Review A 69.6 (2004), p. 062320. DOI: 10.1103/PhysRevA.69.062320.

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[1] Alexandre Blais et al. "Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation". In: Physical Review A 69.6 (2004), p. 062320. DOI: 10.1103/PhysRevA.69.062320.

[2] Jens Koch et al. "Charge-insensitive qubit design derived from the Cooper pair box". In: Physical Review A 76.4 (2007), p. 042319. DOI: 10.1103/PhysRevA.76.042319.